

ESTIMATION OF POWER ATTENUATION OF A NOISY ACOUSTIC CHANNEL

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LUND INSTITUTE OF TECHNOLOGY

Department of Building Acoustics, TVBA-3007.
Telecommunication Theory, Technical Report TR-150.
National Swedish Environment Protection Board,
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ABSTRACT

In this paper the task of measuring attenuation of a noisy channel is studied. A technique that uses on-off modulation of the exciting signal is presented. A discrete time method estimating signal power and background noise power is analysed and shown to be efficient on certain assumptions. A corresponding analog time method is also studied and found to be equivalent to the discrete time method for any reasonable measuring situation.

It is shown that by introducing an idle interval T_p at the detector, the estimates become unbiased and well defined. An implementation is made and an outline of the design is presented. Introductory measurements using this realization are also given.

1. INTRODUCTION

In several acoustic measurements background noise causes a problem resulting in a low signal-to-noise ratio. One way to cope with this problem is by increasing the power of the exciting signal. The power, however, is constrained because an increased input power requires a more powerful exciting device, i.e. in general a more expensive and larger device. Furthermore an increased power gives an increased sound level that might not be tolerated. This means that a measuring method which increases the signal-to-noise ratio without increasing the input power would be most valuable.

In this report we study a method to measure power attenuation of a noisy channel. This attenuation will now be defined. Let us introduce a source, denoted S, that excites the acoustic system with a stationary signal and two points, A and B, in this system. The power attenuation between A and B is then defined by

$$\gamma_{AB} = 10 \log(\overline{p_{SA}^2} / \overline{p_{SB}^2}) \quad (1.1)$$

where $\overline{p_{SA}^2}$ and $\overline{p_{SB}^2}$ are the mean value of the power caused by the source S in the points A and B respectively. This power attenuation is estimated in a number of applications, e.g. in measurement of sound propagation, including sound barriers and in sound and vibration isolation measurements. In the referred situations the background noise often causes low signal-to-noise ratios.

Here the task is to measure the power attenuation of noisy acoustical indoor and outdoor channels. An acoustical indoor channel can with a high degree of accuracy be modelled into a linear and time-invariant system. An acoustical outdoor channel may not in general be modelled in this way. Such a channel is often time-varying and has large phase fluctuations. This means that the well-known coherence method, [1], is not in general applicable, [2]. There is, however, another method suggested in [3] that can be used even for outdoor acoustic measurements. The method comprises a power correlation tech-

nique. This method will here be presented, analysed and developed. In the analysis the time-invariant basis will be used.

The power attenuation of an acoustic channel is in general measured over a number of frequency bands, usually 1/1-octave bands and 1/3-octave bands. This is done by using a wide-band exciting signal, $x_0(t)$, and a band-pass filter according to Fig. 1.1.

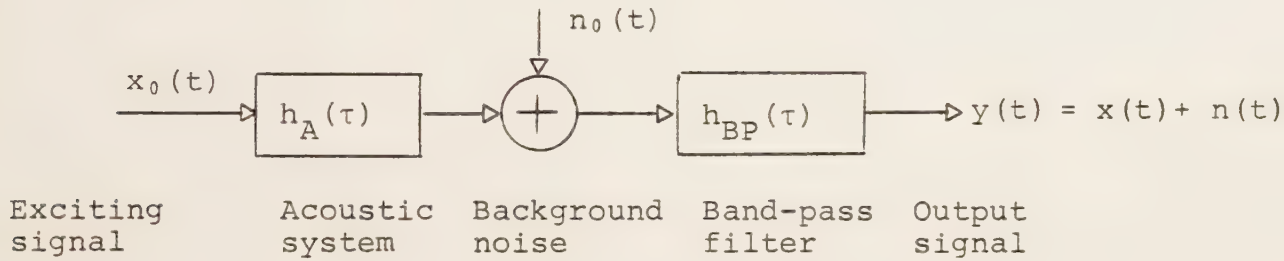


Figure 1.1. The measuring situation.

The impulse responses of the band-pass filter and the acoustic system are denoted $h_{BP}(\tau)$ and $h_A(\tau)$ respectively. The background noise signal is denoted $n_0(t)$. The output signal, $y(t)$, is given by

$$y(t) = x(t) + n(t) \quad (1.2)$$

where

$$x(t) = \int_0^{\infty} h(\tau) x_0(t-\tau) d\tau, \quad (1.3)$$

$$h(t) = \int_0^t h_A(\tau) h_{BP}(t-\tau) d\tau \quad (1.4)$$

and

$$n(t) = \int_0^{\infty} h_{BP}(\tau) n_0(t-\tau) d\tau \quad (1.5)$$

The input signal, $x_0(t)$, is assumed to be a zero-mean white random process with the spectral density $R_0/2$. The power of the signal $x(t)$ is then defined by

$$\sigma_x^2 = E\{x^2(t)\} \quad (1.6)$$

where $E\{\cdot\}$ denotes statistical expectation.

The filtered background signal, $n(t)$, is assumed to be a zero-mean in wide sense stationary random process, statistically independent of $x(t)$. The power of $n(t)$ is

$$\sigma_n^2 = E\{n^2(t)\} \quad (1.7)$$

We find

$$\sigma_x^2 = \frac{R_0}{2} \int_{-\infty}^{\infty} |H(f)|^2 df \quad (1.8)$$

and introduce

$$\sigma_0^2 = \frac{R}{2} \int_{-\infty}^{\infty} |H_{BP}(f)|^2 df \quad (1.9)$$

where $H(f)$ and $H_{BP}(f)$ denote the Fourier transform of $h(\tau)$ and $h_{BP}(\tau)$ respectively. The attenuation that shall be estimated is defined by

$$\gamma = 10[{}^{10}\log\left(\int_{-\infty}^{\infty} |H_{BP}(f)|^2 df\right) - {}^{10}\log\left(\int_{-\infty}^{\infty} |H(f)|^2 df\right)] \quad (1.10)$$

and we note that

$$\gamma = 10[{}^{10}\log \sigma_0^2 - {}^{10}\log \sigma_x^2] \quad (1.11)$$

The quantity σ_0^2 is assumed to be known. This means that by estimating σ_x^2 an estimate of γ can be calculated. The filtered background noise power, σ_n^2 , is also of interest in many applications, e.g. a reference system assisting industrial measurements, [4]. In this paper we therefore will consider the task of estimating both σ_x^2 and σ_n^2 .

The method to be considered will now be introduced. In order to estimate σ_x^2 and σ_n^2 the input signal is switched on and off by a signal $p(t)$ according to Fig. 1.2.

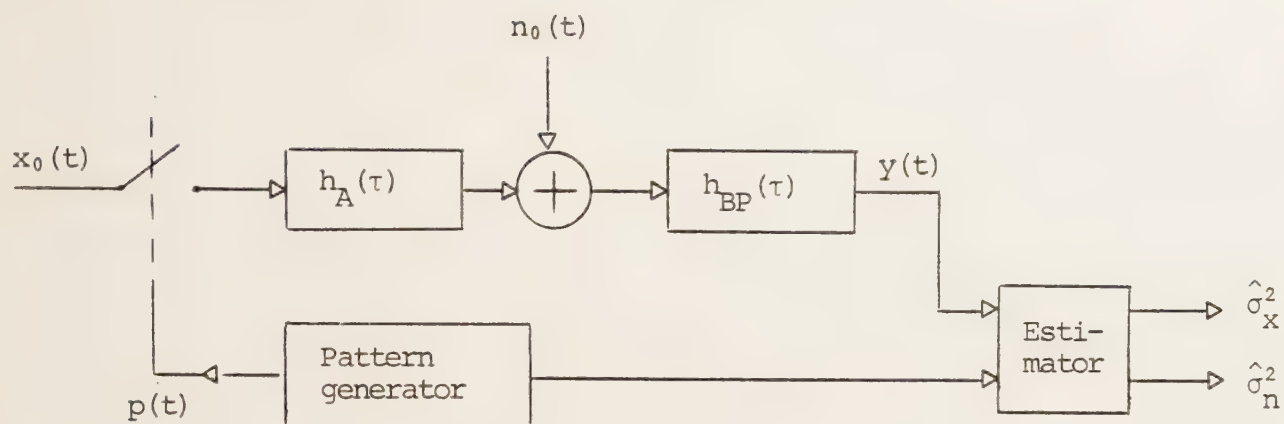


Figure 1.2. The measuring system.

The on-off pattern generator and the estimator, i.e. the power correlator, will be treated in chapter 3. Note that the basic idea of the method is to make alternating measurements; partly of the filtered background signal $n(t)$ and partly of the combined signal $[x(t) + n(t)]$. In chapter 2 this basic idea is analysed in the form of a single on-off measurement using a discrete time approach as well as an analog time approach. A design of a digital power correlator for acoustical measurements is outlined in chapter 4. Measurements using this digital power correlator are presented in chapter 5.

2. A SINGLE ON-OFF MEASUREMENT

2.1 Introduction

The basic idea will here be studied assuming that two independent observations of the output signal $y(t)$ are available. One over a time interval T_1 when the exciting signal is $x_0(t)$, "on", and another over a time interval T_2 when the exciting signal is zero, "off". The assumption is made in order to give a suitable simplified start to the study. The observed output signal $y(t)$ for these two intervals is then $x(t)+n(t)$ and $n(t)$ respectively. Note that this is valid if $p(t) = 1$, "on", a sufficiently long time, T_p , before the time interval T_1 and $p(t) = 0$, "off", a sufficiently long time, T_p before the interval T_2 . The total observation time denoted T_0 , is

$$T_0 = T_1 + T_2 \quad (2.1)$$

According to chapter 1 the task is to estimate σ_x^2 and σ_n^2 from these observations.

We assume that σ_x^2 and σ_n^2 are non random parameters and that the signals $x(t)$ and $n(t)$ are Gaussian processes. It is also assumed that the spectral densities of $x(t)$ and $n(t)$ are

$$R_x(f) = \frac{\sigma_x^2}{2B} I_B^{f_1}(f) \quad (2.2)$$

and

$$R_n(f) = \frac{\sigma_n^2}{2B} I_B^{f_1}(f) \quad (2.3)$$

respectively where $I_B^{f_1}(f)$ is defined by

$$I_B^{f_1}(f) = \begin{cases} 1 & f_1 < |f| < f_1 + B \\ 0 & |f| < f_1 \text{ or } |f| > f_1 + B \end{cases} \quad (2.4)$$

This means that the observed signal is ideally bandwidth limited.

2.2 Discrete time approach

2.2.1 Introduction

Here the observed signal $y(t)$ is sampled and the samples are used to estimate σ_x^2 and σ_n^2 . The sampling procedure is a natural approach since the bandwidth of the observed signal is limited. This approach is also used for designing a digital estimation system, presented in chapter 4. We will, using the assumptions, find maximum likelihood, ML, estimates of σ_x^2 and σ_n^2 . This means that an estimation method is deduced.

The signals $x(t)$ and $n(t)$ are band-pass signals provided that $f_1 \neq 0$, (2.2 - 2.4). In the sampling theorem for band-pass signals the two quadrature components shall be sampled with a sampling rate greater than B . However, if $W_1 = nW$, $W_1 < f_1$ and $f_1 + B \leq W_1 + W$ are satisfied, where n is an integer, the signal itself may be sampled with the rate $f_s = 2W$, [5]. We assume

$$f = n B \quad (2.5)$$

and

$$f_s = 2B \quad (2.6)$$

Let the samples of $y(t)$ when $p(t) = 1$ be $\{y_k\}_{k=1}^{L_1}$, i.e. $\{x_k + n_k\}_{k=1}^{L_1}$ where

$$y_k = y(kT_s) \quad (2.7)$$

$$x_k = x(kT_s) \quad (2.8)$$

$$n_k = n(kT_s) \quad (2.9)$$

$$T_s = 1/f_s \quad (2.10)$$

and L_1 the number of samples. The number of samples when $p(t) = 0$ is L_2 and these are denoted by $\{y_k\}_{k=L_1+1}^{L_0}$, i.e.

$\{n_k\}_{k=L_1+1}^{L_0}$ where L_0 is the total number of samples. We have

$$L_0 = L_1 + L_2 \quad (2.11)$$

The assumptions give

$$E\{y_i y_j\} = c_i \delta_{ij} \quad (2.12)$$

where

$$\delta_{ij} = \begin{cases} 1 & i=j \\ 0 & i \neq j \end{cases} \quad (2.13)$$

and

$$c_i = \begin{cases} \sigma_x^2 + \sigma_n^2 & i = 1, 2, \dots, L_1 \\ \sigma_n^2 & i = L_1 + 1, \dots, L_0 \end{cases} \quad (2.14)$$

2.2.1 ML estimates

The samples, $\{y_k\}_{k=1}^{L_0}$ are Gaussian and, because of (2.12), independent. This means that the likelihood function becomes

$$L(\alpha, \beta) = \left[\prod_{k=1}^{L_0} \frac{1}{\pi(\alpha+\beta)} e^{-\frac{y_k^2}{2(\alpha+\beta)}} \right] \cdot \left[\prod_{k=L_1+1}^{L_0} \frac{1}{\pi\alpha} e^{-\frac{y_k^2}{2\alpha}} \right] \quad (2.15)$$

where

$$\alpha = \sigma_n^2 \quad (2.16)$$

and

$$\beta = \sigma_x^2 \quad (2.17)$$

To find the ML estimates of α and β we must find the absolute minimum of the log likelihood function defined by

$$l(\alpha, \beta) = - \ln[L(\alpha, \beta)] \quad (2.18)$$

The function is given by

$$l(\alpha, \beta) = \frac{A}{\alpha + \beta} + L_1 \ln(\alpha + \beta) + \frac{B}{\alpha} + (L_0 - L_1) \ln(\alpha) \quad (2.19)$$

where

$$A = \sum_{k=1}^{L_1} Y_k^2 = \sum_{k=1}^{L_1} (x_k + n_k)^2 \quad (2.20)$$

and

$$B = \sum_{k=L_1+1}^{L_0} Y_k^2 = \sum_{k=L_1+1}^{L_0} n_k^2 \quad (2.21)$$

The ML estimates, $\hat{\alpha}_{ML}$ and $\hat{\beta}_{ML}$, that minimize $l(\alpha, \beta)$ are found to be

$$\hat{\alpha}_{ML} = \frac{B}{L_0 - L_1} = \frac{1}{L_0 - L_1} \sum_{k=L_1+1}^{L_0} Y_k^2 \quad (2.22)$$

and

$$\hat{\beta}_{ML} = \frac{A}{L_1} - \frac{B}{L_0 - L_1} = \frac{1}{L_1} \sum_{k=1}^{L_1} Y_k^2 - \frac{1}{L_0 - L_1} \sum_{k=L_1+1}^{L_0} Y_k^2 \quad (2.23)$$

This is a very simple estimation procedure. We note that the estimates are unbiased. By means of [6] and [7] we also observe that the estimation procedure makes an efficient two-parameter estimate.

The variance of $\hat{\beta}_{ML}$ is

$$V\{\hat{\beta}_{ML}\} = E\{(\hat{\beta}_{ML} - \beta)^2\} = 2\left[\frac{(\alpha + \beta)^2}{L_1} + \frac{\alpha^2}{L_0 - L_1}\right] \quad (2.24)$$

and of $\hat{\beta}_{ML}$

$$V\{\hat{\beta}_{ML}\} = \{E(\hat{\alpha}_{ML} - \alpha)^2\} = \frac{2\alpha^2}{L_0 - L_1} \quad (2.25)$$

Let us assume that the total number of samples, L_0 , is fixed and that L_1 may be chosen in the interval $0 < L_1 < L_0$ in order to minimize the variance of $\hat{\beta}_{ML}$. The optimum value of L_1 , denoted L_1^0 , is then given by

$$L_1^0 = L_0 \left(\frac{\alpha + \beta}{2\alpha + \beta} \right) = L_0 \left(\frac{\sigma_n^2 + \sigma_x^2}{2\sigma_n^2 + \sigma_x^2} \right) = L_0 \left(\frac{s_{NR} + 1}{s_{NR} + 2} \right) \quad (2.26)$$

where s_{NR} denotes the signal-to-noise ratio defined by

$$s_{NR} = \frac{\sigma_x^2}{\sigma_n^2} \quad (2.27)$$

The corresponding value of the variance i.e. the minimum becomes

$$V_{L_1^0} \{\hat{\beta}_{ML}\} = \frac{2}{L_0} (2\alpha + \beta)^2 = \frac{2}{L_0} (2\sigma_n^2 + \sigma_x^2)^2 \quad (2.28)$$

If there is some rough a priori knowledge of the signal-to-noise ratio, s_{NR} , one might use (2.26) in order to find a suitable value of L_1 . In any case we note that

$$\frac{1}{2} < \frac{L_1^0}{L_0} < 1 \quad (2.29)$$

2.2.3 Comments from a practical point of view

The 1/1-octave filters and also the 1/3-octave filters, used in practical measurements, match (2.6) fairly well. For 1/1-octave filters we have

$$f_1 = B \quad (2.30)$$

and

$$f_0 = \sqrt{2} B \quad (2.31)$$

that is the centre frequency, the geometric mean.

For 1/3-octave filters we note that

$$f_1 = \frac{1}{2^{1/3} - 1} B \simeq 4 B \quad (2.32)$$

and

$$f_0 = f_1 2^{1/6} = \frac{2^{1/6}}{2^{1/3} - 1} B \quad (2.33)$$

This means that if the input signal fed into the band-pass filter, 1/1-octave or 1/3-octave, is stationary and white relative to the filter, formula (2.12) will be satisfied to a high degree of accuracy provided that $f_s = 2 B$.

This gives

$$f_s = 2 \frac{1}{\sqrt{2}} f_0 = \sqrt{2} f_0 \simeq \frac{3}{2} f_0 \quad (2.34)$$

for 1/1-octave filters and

$$f_s = 2 \frac{2^{1/3} - 1}{2^{1/6}} f_0 \simeq \frac{f_0}{2} \quad (2.35)$$

for 1/3-octave filters.

The estimates given in (2.22) and (2.23) are robust. For example if (2.12) is not valid the estimates will still be

unbiased and usable. This means that the choice of sampling rate is not critical.

We will also comment (2.26). In difficult measuring situations we have $s_{NR} \ll 1$ giving $L_1^0 \simeq \frac{L_0}{2}$. A natural choice of a fixed L_1 is therefore

$$L_1 = \frac{L_0}{2} \quad (2.36)$$

2.3 Analog time approach

In this section we will not deduce a method but be content with only suggesting an analog method. This approach is used in [3]. The suggested method corresponds to the deduced discrete time method. Throughout we will also use the corresponding notations. The analog estimates are given by

$$\hat{\alpha} = \frac{1}{T_2} \int_{T_p + T_1}^{T_p + T_1 + T_2} y^2(t) dt = \frac{1}{T_2} \int_{T_p + T_1}^{T_p + T_1 + T_2} n^2(t) dt \quad (2.37)$$

and

$$\begin{aligned} \hat{\beta} &= \frac{1}{T_1} \int_0^{T_1} y^2(t) dt - \frac{1}{T_2} \int_{T_p + T_1}^{T_p + T_1 + T_2} y^2(t) dt = \\ &= \frac{1}{T_1} \int_0^{T_1} (x(t) + n(t))^2 dt - \frac{1}{T_2} \int_{T_p + T_1}^{T_p + T_1 + T_2} n^2(t) dt \quad (2.38) \end{aligned}$$

where the transmitted signal is $x_0(t)$ for $t \leq T_1$ and zero for $t > T_1$. We choose T_p large enough to be able to assume $y(t)$, for $t \geq T_p + T_1$, to be a wide-sense stationary random process independent of $y(t)$ for $t \leq T_1$ this analog time method will now be analysed and compared with the discrete time method.

We observe that the estimates are unbiased since

$$E\{\hat{\alpha}\} = \sigma_n^2 \quad (2.39)$$

and

$$E\{\hat{\beta}\} = \sigma_x^2 \quad (2.40)$$

The variances are given by

$$\begin{aligned} V\{\hat{\beta}\} &= E \left\{ \frac{1}{T_1^2} \int_0^{T_1} \int_0^{T_1} (x(t_1) + n(t_1))^2 (x(t_2) + n(t_2))^2 dt_1 dt_2 + \right. \\ &\quad + \frac{1}{T_2^2} \int_{T_p+T_1}^{T_p+T_1+T_2} \int_{T_p+T_1}^{T_p+T_1+T_2} n^2(t_1) n^2(t_2) dt_1 dt_2 - \\ &\quad \left. - \frac{2}{T_1 T_2} \int_0^{T_1} \int_{T_p+T_1}^{T_p+T_1+T_2} (x(t_1) + n(t_1))^2 n^2(t_2) dt_1 dt_2 \right\} - \sigma_x^4 = \\ &= \frac{2}{T_1^2} \int_0^{T_1} \int_0^{T_1} [r_x^2(t_1 - t_2) + 2r_n(t_1 - t_2)r_x(t_1 - t_2) + \\ &\quad + r_n^2(t_1 - t_2)] dt_1 dt_2 + \frac{2}{T_2^2} \int_0^{T_2} \int_0^{T_2} r_n^2(t_1 - t_2) dt_1 dt_2, \end{aligned} \quad (2.41)$$

and

$$V\{\hat{\alpha}\} = \frac{2}{T_2^2} \int_0^{T_2} \int_0^{T_2} r_n^2(t_1 - t_2) dt_1 dt_2 \quad (2.42)$$

where $r_n(\tau)$ and $r_x(\tau)$ are covariance functions of the noise and the signal component of the received signal $y(t)$ respectively. Since the spectral densities of these signals are assumed to be constant over the frequency interval $f_1 < |f| < f_1 + B$ the variances are upper bounded by

$$V\{\hat{\beta}\} < \frac{(\sigma_x^2 + \sigma_n^2)^2}{BT_1} + \frac{\sigma_n^4}{BT_2} \quad (2.43)$$

and

$$V\{\hat{\alpha}\} < \frac{\sigma_n^4}{BT_2} \quad (2.44)$$

For $BT \gg 1$ and $BT \gg 1$ it is found that

$$V\{\hat{\beta}\} \simeq \frac{(\sigma_x^2 + \sigma_n^2)^2}{BT_1} + \frac{\sigma_n^4}{BT_2} \quad (2.45)$$

and

$$V\{\hat{\alpha}\} \simeq \frac{\sigma_n^4}{BT_2} \quad (2.46)$$

These formulas are derived in appendix.

In order to compare these analog estimates with the discrete time estimates we prescribe the same estimation time and use (2.6) giving

$$T_s = \frac{1}{f_s} = \frac{1}{2B} \quad (2.47)$$

We then find for $BT_1 \gg 1$ and $BT_2 \gg 1$ that the discrete time variances, (2.24) and (2.25), equal the analog time variances.

We now assume that (2.45) is a valid approximation and that the total time, T_0 , is fixed and that T_1 may be chosen in the interval $0 < T_1 < T_0$ in order to minimize the variance of the estimate $\hat{\beta}$. The optimum value of T_1 , denoted T_1^O , is then given by

$$T_1^O = T_0 \cdot \left(\frac{\sigma_n^2 + \sigma_x^2}{2\sigma_n^2 + \sigma_x^2} \right) \quad (2.48)$$

The corresponding value of the variance, that is the minimum, becomes

$$\text{Var}_{T_0} \{\hat{\beta}\} = \frac{1}{BT_0} (2\sigma_n^2 + \sigma_x^2)^2 \quad (2.49)$$

This may be used if there is some a priori knowledge of the signal-to-noise ratio σ_x^2/σ_n^2 . In any case we note that

$$\frac{1}{2} < \frac{T_1^0}{T_0} < 1. \quad (2.50)$$

We record that this is equivalent to (2.29). Using the comments given in section 2.2.3 of this paper we note that a natural choice is

$$T_1 = \frac{T_0}{2} \quad (2.51)$$

2.4 Deterministic exciting signal

An often used technique is to replace the exciting random signal by a deterministic pseudorandom signal. Let us assume that $x_0(t)$ is a pseudorandom signal with the period $T_1 = L_1 T$ generated in a suitable way using a binary maximum-length sequence such that

$$\beta_1 = \frac{1}{T_1} \int_0^{T_1} x^2(t) dt \simeq T_s \sum_{k=1}^{L_1} x_k^2 \quad (2.52)$$

and

$$\gamma = 10[{}^{10}\log \beta_0 - {}^{10}\log \beta_1] \quad (2.53)$$

where β_0 is a known constant.

More details will be given in chapter 4. We prescribe $T_1 = T_2$, i.e. $L_1 = L_2$. Using $\hat{\beta}_{ML}$, as an estimate of β , gives

$$E\{\hat{\beta}_{ML}\} = \beta_1 \quad (2.54)$$

and the variance

$$V\{\hat{\beta}_{ML}\} = \frac{4\sigma_n^4 + 4\sigma_n^2 \beta_1}{L_1} = \frac{2\sigma_n^4 + 2\sigma_n^2 \beta_1}{BT_1} = \frac{2\alpha^2 + 2\alpha\beta}{BT_1} \quad (2.55)$$

This technique is therefore most valuable in measuring situations where the signal-to-noise ratio is high. In other situations the gain of using this input signal is low. If the generator is appropriately designed, there are however no essential drawbacks. Therefore this generator may be considered as a possible and simple way to realize a suitable exciting signal.

3. PATTERN MODULATED MEASUREMENTS

3.1. The power correlation method

The motive for introducing pattern modulated measurements is that the acoustic system is unknown or almost unknown. This means that we don't know the duration or the delay of its impulse response. In such a situation it is very difficult, using a limited measuring time, to guarantee stationary signals and independent measurements as assumed in chapter 2. We will here however show that this situation can be handled using pattern modulated measurements. Since the discrete time approach and the analog approach are equivalent provided that $f_s = 2B$, $BT_1 \gg 1$ and $BT_2 \gg 1$, we will in this chapter be content with using only one approach. In [3] the analog approach is used giving an estimation method. In order to aid comparison between this method and the one suggested in this paper we choose to use the analog time approach.

First we recall from chapter 1 the measuring system. This is illustrated, showing more details, in Fig. 3.1.

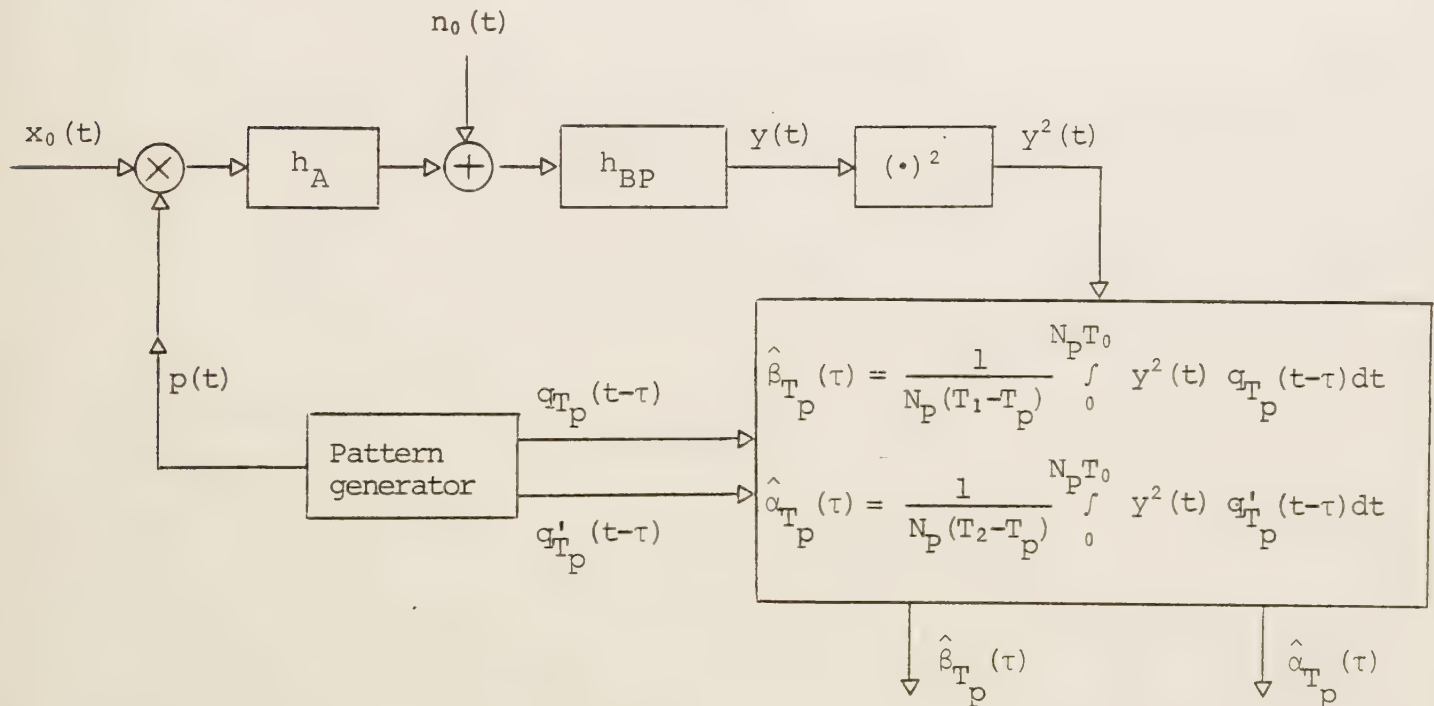


Figure 3.1. The measuring system using pattern modulation, i.e. the power correlator.

First we assume that the pattern switch signal, $p(t)$, is a periodic signal given by

$$p(t+lT_0) = \begin{cases} 1 & 0 \leq t < T_1 \\ 0 & T_1 \leq t < T_0 \end{cases} \quad (3.1)$$

where l is an integer. In chapter 2 it was found that a suitable fixed value of T_1 is

$$T_1 = T_2 = T = \frac{T_0}{2} \quad (3.2)$$

This relation will now be valid throughout the paper. The pattern correlation signals are also periodic signals and defined by

$$q_{T_p}(t+lT_0) = \begin{cases} 1 & T_p \leq t < T \\ 0 & 0 \leq t < T_p \text{ and } T \leq t < T+T_p \\ -1 & T+T_p \leq t < T_0 \end{cases} \quad (3.3)$$

and

$$q'_{T_p}(t+lT_0) = \begin{cases} 1 & T+T_p \leq t < T_0 \\ 0 & 0 \leq t < T+T_p \end{cases} \quad (3.4)$$

where

$$T_p < T \quad (3.5)$$

We note that

$$q_0(t) = 2 \cdot p(t) - 1 \quad (3.6)$$

and

$$q'_0(t) = 1 - p(t) \quad (3.7)$$

Let us study the estimating function, $\hat{\beta}_{T_p}(\tau)$, given by

$$\hat{\beta}_{T_p}(\tau) = \frac{1}{N_p(T-T_p)} \int_0^{N_p T_0} y^2(t) q_{T_p}(t-\tau) dt \quad (3.8)$$

where N is an integer.

We note that $\hat{\beta}_{T_p}(\tau)$ is a periodic correlation function with the period T_0 . The mean value of $\hat{\beta}_{T_p}(\tau)$ is

$$E\{\hat{\beta}_{T_p}(\tau)\} = \frac{1}{N_p(T-T_p)} \int_0^{N_p T_0} \sigma_y^2(t) q_{T_p}(t-\tau) dt \quad (3.9)$$

where

$$\sigma_y^2(t) = E\{y^2(t)\}. \quad (3.10)$$

We now assume that the system $h(\tau)$ comprises a delay, T_d , and has a finite duration, T_D , of its impulse response. This means that for $\tau < T_d$ and for $\tau > T_d + T_D$ we have $h(\tau) = 0$. In practice the assumption can always be met with a high degree of accuracy for suitable values of T_d and T_D . Let us, using these assumptions, plot $q_{T_p}(t)$ and in point of principle also $\sigma_y^2(t)$ and $E\{\hat{\beta}_{T_p}(\tau)\}$. This is done in Fig. 3.2.

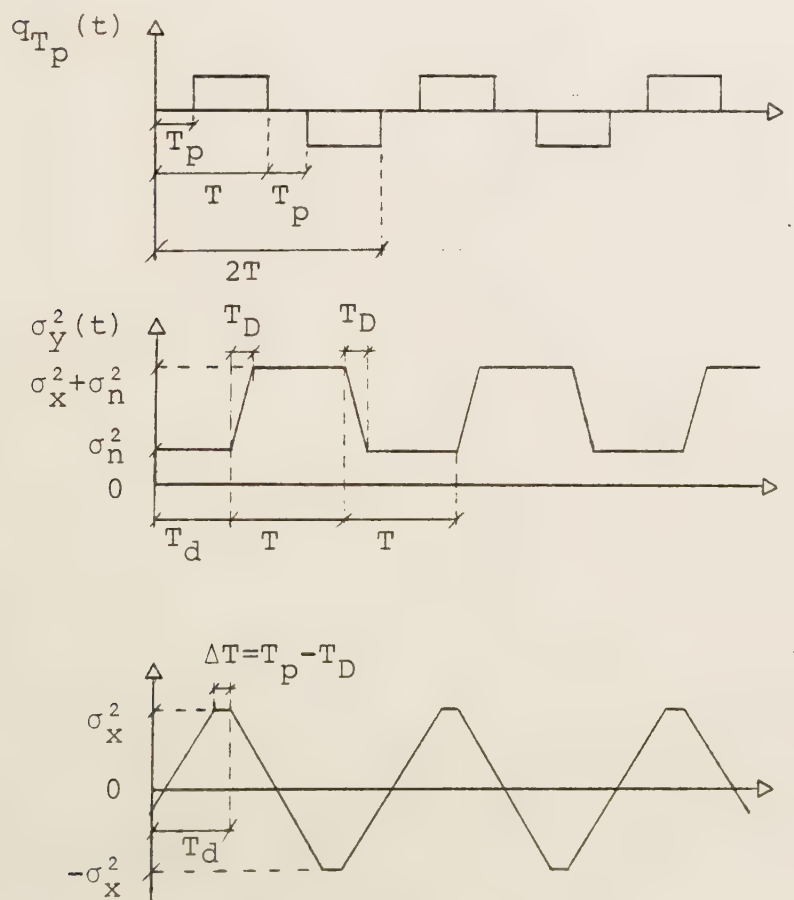


Figure 3.2. The function $q_{T_p}(t)$ and in point of principle $\sigma_y^2(t)$ and $E\{\hat{\beta}_{T_p}(\tau)\}$.

The function $\sigma_Y^2(t)$ is periodic and for $0 \leq t \leq T_D$ we have

$$\begin{aligned}\sigma_Y^2(t+nT_0+T_d) &= \sigma_n^2 + \int_0^t \int_0^t h(\tau_1) h(\tau_2) r_{x_0}(\tau_1-\tau_2) d\tau_1 d\tau_2 = \\ &= \sigma_n^2 + \frac{R_0}{2} \int_0^t h^2(\tau) d\tau\end{aligned}\quad (3.11)$$

since the spectral density of $x_0(t)$ is $R_0/2$. For $T_D \leq t \leq T$ we have

$$\sigma_Y^2(t+nT_0+T_d) = \sigma_n^2 + \sigma_x^2 \quad (3.12)$$

since

$$\sigma_x^2 = \frac{R_0}{2} \int_0^{T_D} h^2(\tau) d\tau = \frac{R_0}{2} \int_{-\infty}^{\infty} |H(f)|^2 df. \quad (3.13)$$

For $T \leq t \leq T+T_D$ we have

$$\begin{aligned}\sigma_Y^2(t+nT_0+T_d) &= \sigma_n^2 + \int_{t-T}^{T_D} \int_{t-T}^{T_D} h(\tau_1) h(\tau_2) r_{x_0}(\tau_1-\tau_2) d\tau_1 d\tau_2 = \\ &= \sigma_n^2 + \frac{R_0}{2} \int_{t-T}^{T_D} h^2(\tau) d\tau\end{aligned}\quad (3.14)$$

and for $T+T_D \leq t \leq T_0$

$$\sigma_Y^2(t+nT_0+T_d) = \sigma_n^2 \quad (3.15)$$

One observes that, provided that

$$\Delta T = T_p - T_D > 0, \quad (3.16)$$

the maximum of $E\{\hat{\beta}_{T_p}(\tau)\}$ becomes σ_x^2 over an interval ΔT_0 . This is an important result. If (3.16) is not valid there exists no such interval and the maximum of $E\{\hat{\beta}_{T_p}(\tau)\}$ becomes a biased estimate of σ_x^2 . We therefore suggest the following handy method.

One increases the value of N until the estimating function $\hat{\beta}_{T_p}(\tau)$ becomes a good estimate of $E\{\hat{\beta}_{T_p}(\tau)\}$. In practice this means that N_p is doubled until $\hat{\beta}_{T_p}(\tau)$ for N_p equals or almost equals $\hat{\beta}_{T_p}(\tau)$ for $N_p/2$. If then there is an interval ΔT , in which $\hat{\beta}_{T_p}(\tau)$ is constant or almost constant and maximum, one has found an estimate of σ_x^2 . We denote this estimate $\hat{\beta}_{T_p}$. If there exists no such interval, T_p has to be increased and $E\{\hat{\beta}_{T_p}(\tau)\}$ estimated using the new value of T_p and so on. The estimating function

$$\hat{\alpha}_{T_p}(\tau) = \frac{1}{N_p(T-T_p)} \int_0^{N_p T_0} y^2(t) q'_{T_p}(t-\tau) dt \quad (3.17)$$

is used in a corresponding way in order to estimate σ_n^2 . The minimum of $\hat{\alpha}_{T_p}(\tau)$ gives the estimate $\hat{\alpha}_{T_p}$. The minimum of $\hat{\alpha}_{T_p}(\tau)$ corresponds to the maximum of $\hat{\beta}_{T_p}(\tau)$.

The definition of T_D provides that

$$T_D \gg \frac{1}{B} \quad (3.18)$$

This means that the estimates become unbiased and the variances are

$$V\{\hat{\beta}_{T_p}\} \simeq \frac{(\sigma_x^2 + \sigma_n^2)^2 + \sigma_n^4}{N_p B(T-T_p)} \quad (3.19)$$

and

$$V\{\hat{\alpha}_{T_p}\} \simeq \frac{\sigma_n^4}{N_p B(T-T_p)} \quad (3.20)$$

respectively, provided that $B(T-T_p) \gg 1$.

We close this section by noting that from the estimating functions $\hat{\beta}_{T_p}(\tau)$ and $\hat{\alpha}_{T_p}(\tau)$ one obtains good estimates of σ_x^2 and σ_n^2 and that the problems caused by unknown delay and duration are solved by using the maximum of this correlation function and an appropriate pause T_p respectively. This means also that the delay, T_d , and the duration, T_D , become in some sense estimated.

3.2 The power correlation method without pauses

The method suggested in [3] does not use any pauses, i.e. $T_p = 0$. We will therefore in this section analyse this situation assuming $T_D < T$. Since in [3] only σ_x^2 is estimated we restrict the analysis here to estimation of σ_x^2 . The estimation procedure is given in 3.1. We find that the estimate, $\hat{\beta}_0$, is in general biased and note that

$$1 - \frac{2T_D}{T} \leq E\{\hat{\beta}_0\}/\sigma_x^2 \leq 1 \quad (3.21)$$

This is due to the fact that for $0 \leq t \leq T_D$ and $T < t < T+T_D$ we have

$$\sigma_n^2 \leq \sigma_y^2(t) \leq \sigma_n^2 + \sigma_x^2 \quad (3.22)$$

The variance of $\hat{\beta}_0$ is calculated in [7]. It is found that

$$\begin{aligned} V\{\hat{\beta}_0\} &\leq \frac{\sigma_x^4 + 2\sigma_x^2\sigma_n^2 + 2\sigma_n^4}{N_p} \left(\frac{1}{BT} + 18\left(\frac{T_D}{T}\right)^2 \right) = \\ &= \frac{\sigma_x^4 + 2\sigma_x^2\sigma_n^2 + 2\sigma_n^4}{N_p BT} \left(1 + 18BT_D \cdot \frac{T_D}{T} \right) \end{aligned} \quad (3.23)$$

This means that for

$$\frac{T}{2T_D} \gg 1 \quad (3.24)$$

the estimate is almost unbiased and for

$$\frac{T}{2T_D} \gg 9BT_D \quad (3.25)$$

the variance of the estimate $\hat{\beta}_0$ equals the variance of the estimate $\hat{\beta}_{T_p}$. When using this method one therefore has to increase the value of N_p and T until the maximum of the function $\hat{\beta}_0(\tau)$ is well defined. Let us compare this method with the one given in section 3.1 above. The most significant difference

is that when using $T_p=0$ one has to choose $T \gg T_D$ and a more complex procedure to find the maximum of the correlation function. We therefore note that the power correlation method using pauses is in general preferable.

3.3 Some further comments from a practical point of view

3.3.1 Estimation of the background noise

The power of the filtered background noise, σ_n^2 , is estimated by means of $\hat{\alpha}_{T_p}(\tau)$. This means that two estimating functions have to be calculated. We want to avoid this and only calculate one of them. In order to do so we note that

$$\xi = E\left\{\frac{1}{N_P T_0} \int_0^{N_P T_0} y^2(t) dt\right\} = \frac{1}{T_0} \int_0^{T_0} \sigma_Y^2(t) dt \quad (3.26)$$

and that

$$\sigma_Y^2(t + nT_0 + T_d) + \sigma_Y^2(t + nT_0 + T + T_d) = 2\sigma_n^2 + \sigma_x^2 \quad (3.27)$$

This gives

$$\xi = \sigma_n^2 + \frac{1}{2} \sigma_x^2 \quad (3.28)$$

which may be considered as the mean power of $y(t)$. We then design a new estimate of σ_n^2 given by

$$\hat{\alpha}_{T_p}' = \hat{\xi} - \hat{\beta}_{T_p} \quad (3.29)$$

where

$$\hat{\xi} = \frac{1}{N_P T_0} \int_0^{N_P T_0} y^2(t) dt \quad (3.30)$$

This new estimate $\hat{\alpha}_{T_p}'$ is unbiased and since $\hat{\xi}$ is only a point estimate the procedure of estimating σ_x^2 and σ_n^2 is essentially simplified.

3.3.2 On the choice of the pause time

We have in section 3.1 noted that the so called maximum interval of $E\{\hat{\beta}_{T_p}(\tau)\}$ has the length

$$\Delta T = T_p - T_D > 0 \quad (3.31)$$

that is

$$T_p > T_D. \quad (3.32)$$

This is not sufficient if the maximum interval shall be easy to identify. In practice $\hat{\beta}_{T_p}(\tau)$ is calculated over one period and given, as will be shown in chapter 4, in only a number of points. That is, the correlator gives the series $\{\hat{\beta}_{T_p}(kT_c)\}_{k=1}^{K_{T_0}}$, where T_c is the incremental time step between the points such that

$$K_{T_0} = \frac{T_0}{T_c} \quad (3.33)$$

becomes an integer.

In order to guarantee a maximum interval, i.e. $\hat{\beta}_{T_p}(kT_c)$ maximum for at least two points close to each other, one requires that

$$\Delta T = T_p - T_D \geq 2T_c \quad (3.34)$$

since the delay T_d is unknown. The increment T_c gives the resolution of the estimating function $\hat{\beta}_{T_p}(\tau)$. If one is not interested in details of $\hat{\beta}_{T_p}(\tau)$ but only the maximum interval the following is reasonable:

$$T_c \geq \frac{1}{2} T_D \quad (3.35)$$

and thereby

$$T_p \geq 2T_D. \quad (3.36)$$

On the other hand T_p causes a loss of effective estimation time. If the maximum tolerable loss is 50% one obtains

$$T \geq 2T_p \quad (3.37)$$

If one also considers that the background noise might be slowly time-varying the following proposition can be used:

$$T = 2T_p = 4T_D \quad (3.38)$$

which gives a maximum interval

$$\Delta T = T_D = T_p/2. \quad (3.39)$$

3.3.3 The duration of the impulse response

We have several times in this paper used T_D , the duration of the impulse response. We will here show that this quantity for indoor acoustic channels is related to the reverberation time. For an indoor acoustic channel the following relation is valid.

$$\frac{1}{h_0^2} \int_0^\infty h^2(\tau) d\tau \simeq 10^{-6} \frac{t}{T_R} \quad (3.40)$$

where

$$h_0^2 = \int_0^\infty h^2(\tau) d\tau \quad (3.41)$$

and T_R the reverberation time. We note that the duration of the impulse response is related to T_R .

One way to define T_D is

$$\int_{T_D}^\infty h^2(\tau) d\tau = \frac{h_0^2}{100} \quad (3.42)$$

and here this definition gives

$$T_D \simeq T_R/3 \quad (3.43)$$

Provided that $T > T_R/3$, we note that for $0 \leq t \leq T$ we have

$$\sigma_Y^2(t + nT_0 + T_d) \simeq \sigma_n^2 + \sigma_x^2 (1 - 10^{-6} \frac{t}{T_R}) \quad (3.44)$$

and for $T \leq t \leq T_0$

$$\sigma_Y^2(t + nT_0 + T_d) \simeq \sigma_n^2 + \sigma_x^2 10^{-6} \frac{(t-T)}{T_R} \quad (3.45)$$

Generally we have

$$\sigma_Y^2(t) = \sigma_n^2 + \frac{R_0}{2} \int_0^\infty h^2(\tau) p(t - \tau) d\tau \quad (3.46)$$

and this means that $h^2(\tau)$ may be considered as "power modulation response". In room acoustics one has the concept complex modulation transfer function defined by

$$H_M(f) = \int_0^\infty h^2(\tau) e^{-i2\pi f\tau} d\tau \quad (3.47)$$

for the low frequency band, 0-20 Hz.

Using (3.40) it is possible to calculate the function $E\{\hat{\beta}_{T_p}(\tau)\}$. Here we will be content with calculating $E\{\hat{\beta}_{T_p}(\tau)\}$ for some values of τ , provided that $T \geq T_R/3$. Let us start with $E\{\hat{\beta}_0(T_d)\}$. We have

$$E\{\hat{\beta}_0(T_d)\} \simeq \sigma_x^2 \left[\left(\frac{1}{T} \int_0^T (1 - 2 \cdot 10^{-6} \frac{t}{T_R}) dt \right) \right] \simeq \sigma_x^2 \left(1 - \frac{T_R}{6.9T} \right) \quad (3.48)$$

This formula will be used in chapter 5.

We also calculate $E\{\hat{\beta}_{T_p}(T_d)\}$, i.e.

$$\begin{aligned} E\{\hat{\beta}_{T_p}(T_d)\} &\simeq \sigma_x^2 \left[\frac{1}{T - T_p} \int_{T_p}^T (1 - 2 \cdot 10^{-6} \frac{t}{T_R}) dt \right] \simeq \\ &\simeq \sigma_x^2 \left[1 - \frac{T_R}{(T - T_p)} \frac{1}{3 \ln 10} 10^{-6} \frac{T_p}{T_R} (1 - 10^{-6} \frac{(T - T_p)}{T_R}) \right], \end{aligned} \quad (3.49)$$

and $E\{\hat{\beta}_{T_p}(T_d - \Delta)\}$ for $\Delta \leq T_p$, i.e.

$$\begin{aligned} E\{\hat{\beta}_{T_p}(T_d - \Delta)\} &\approx \sigma_x^2 \left[\frac{1}{T - T_p} \int_{T_p - \Delta}^{T - \Delta} (1 - 2 \cdot 10^{-6} \frac{t}{T_R}) dt \right] = \\ &= \sigma_x^2 \left[1 - \frac{T_R}{(T - T_p)} \frac{1}{3 \ln 10} 10^{-6} \frac{T_p - \Delta}{T_R} (1 - 10^{-6} \frac{T - T_p}{T_R}) \right] \end{aligned} \quad (3.50)$$

Let us test the proposition

$$T = 2T_p = 4T_D = 4 \frac{T_R}{3} \quad (3.51)$$

We obtain

$$E\{\hat{\beta}_{T_p}(T_d)\} \approx \sigma_x^2 \left[1 - \frac{10^{-3}}{2 \ln 10} \right] \quad (3.52)$$

and for $\Delta = T_p/2$

$$E\{\hat{\beta}_{T_p}(T_d - \Delta)\} \approx \sigma_x^2 \left[1 - \frac{10^{-2}}{2 \ln 10} \right] \quad (3.53)$$

which is two points of the maximum interval. This means that the proposition worked well.

3.3.4 Normalized standard deviation

We will comment the variance formula

$$V\{\hat{\alpha}_{T_p}\} \approx \frac{\sigma_n^4}{N_p B(T_1 - T_p)} \quad (3.54)$$

and

$$V\{\hat{\beta}_{T_p}\} \approx \frac{(\sigma_x^2 + \sigma_n^2)^2}{N_p B(T_1 - T_p)} + \frac{\sigma_n^4}{N_p B(T_1 - T_p)} \quad (3.55)$$

assuming

$$T = T_1 = T_2 \quad (3.56)$$

The normalized standard deviation of the estimate $\hat{\alpha}_{T_p}$ is defined by

$$\delta_{\alpha}^{\wedge} = \frac{\sqrt{V\{\hat{\alpha}_{T_p}\}}}{E\{\hat{\alpha}_{T_p}\}} \quad (3.57)$$

and we find

$$\delta_{\alpha}^{\wedge} = \frac{1}{\sqrt{N_p B(T-T_p)}} \quad (3.58)$$

Note that for $\delta_{\alpha}^{\wedge} \ll 1$ the standard deviation is easily translated into dB, which is the most common unit in acoustics. Let us assume that a normalized standard deviation value of 0.1 is tolerable, i.e. 0.5 dB. One gets the requirement

$$N_p B(T-T_p) \geq 100. \quad (3.59)$$

The normalized standard deviation of $\hat{\beta}_{T_p}$ is

$$\delta_{\beta}^{\wedge} = \left[\frac{\sigma_x^4 + 2\sigma_x^2 \sigma_n^2 + 2\sigma_n^4}{\sigma_x^4 N_p B(T-T_p)} \right]^{1/2} = \frac{1}{\sqrt{N_p B(T-T_p)}} \sqrt{1 + \frac{2}{S_{NR}} + \frac{2}{S_{NR}^2}} \quad (3.60)$$

where

$$S_{NR} = \frac{\sigma_x^2}{\sigma_n^2} \quad (3.61)$$

We observe that for $S_{NR} \gg 1$, $\delta_{\beta}^{\wedge}$ becomes

$$\delta_{\beta}^{\wedge} \approx \frac{1}{\sqrt{N_p B(T-T_p)}} \quad (3.62)$$

and for $S_{NB} \ll 1$

$$\delta_{\beta}^{\wedge} \approx \frac{1}{S_{NR}} \frac{\sqrt{2}}{\sqrt{N_p B(T-T_p)}} = \frac{\sigma_n^2}{\sigma_x^2} \frac{\sqrt{2}}{\sqrt{N_p B(T-T_p)}} \quad (3.63)$$

By means of these formulas the estimation time required for a given normalized standard deviation can be calculated. Conversely the normalized standard deviation can be calculated for a given estimation time.

3.4 Estimation with a pseudorandom pattern signal

In [3] the pattern signal is generated by means of a binary maximum-length sequence. We briefly describe and analyse this situation.

Let $\{p_k\}_{k=-\infty}^{\infty}$ be a binary (0 or 1) periodic sequence with the period $N_0 = 2^n - 1$ generated by an n -stage maximum-length shift register. We will use a few qualities of the sequence $\{p_k\}_{k=-\infty}^{\infty}$, found for example in [8].

Let us introduce

$$q_k = 2p_k - 1. \quad (3.64)$$

Then we have

$$c_\ell = \sum_{k=1}^{N_0} p_k q_{k+\ell} = \begin{cases} \frac{N_0+1}{2} & \text{for } \ell = 0, \pm N_0, \pm 2N_0, \dots \\ 0 & \text{otherwise} \end{cases} \quad (3.65)$$

and

$$\sum_{k=1}^{N_0} q_k = 1 \quad (3.66)$$

The pattern signal is defined by

$$p(t) = \sum_{k=-\infty}^{\infty} p_k a_0(t-kT) \quad (3.67)$$

where

$$a_0(t) = \begin{cases} 1 & 0 < t < T \\ 0 & \text{otherwise} \end{cases} \quad (3.68)$$

The pattern signal is periodic with the period time

$$T_0 = N_0 T. \quad (3.69)$$

The pattern correlation signal is

$$q_{T_p}(t) = \sum_{k=-\infty}^{\infty} q_k a_{T_p}(t-kT) \quad (3.70)$$

where

$$a_{T_p}(t) = \begin{cases} 1 & T_p < t < T \\ 0 & \text{otherwise} \end{cases} \quad (3.71)$$

and the correlation function $r_{T_p}(\tau)$ is

$$r_{T_p}(\tau) = \frac{2}{(T-T_p)(N_0+1)} \int_0^{N_0 T} p(t) q_{T_p}(t-\tau) d\tau \quad (3.72)$$

The function is plotted in Fig. 3.3 for $N=7$.

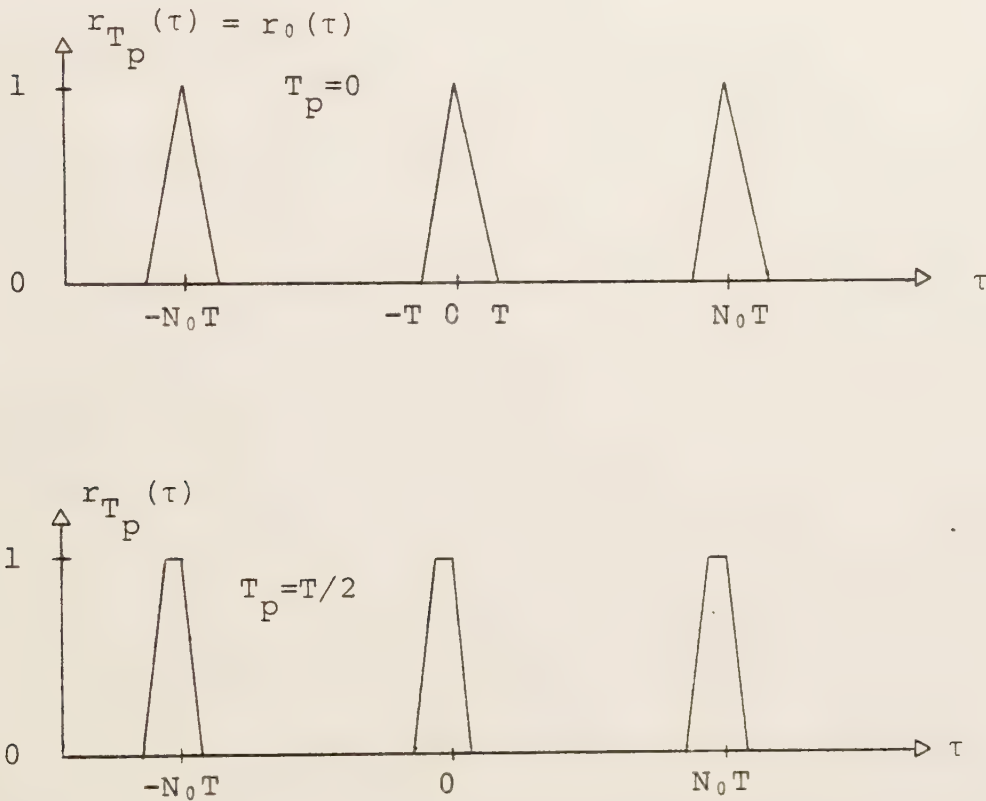


Figure 3.3. The correlation function $r_{T_p}(\tau)$ for $T_p=0$ and $T_p=T/2$ and $N=7$.

The estimate of σ_x^2 , used in [3], is generated by the estimating function

$$\hat{\beta}_0(\tau) = \frac{2}{T(N_0+1)} \int_0^{N_0 T} y^2(t) q_0(t-\tau) dt \quad (3.73)$$

The mean value of $\hat{\beta}_0(\tau)$ is

$$\begin{aligned} E\{\hat{\beta}_0(\tau)\} &= \frac{2}{(N_0+1)} \sigma_n^2 + \frac{R_0}{2} \int_0^\infty \int_0^{N_0 T} h^2(\sigma) p(t-\sigma) q_0(t-\tau) d\sigma dt = \\ &= \frac{2}{(N_0+1)} \sigma_n^2 + \frac{R_0}{2} \int_0^\infty h^2(\sigma) r_0(\sigma-\tau) d\sigma. \end{aligned} \quad (3.74)$$

Provided that $T_D \ll T$ we have

$$E\{\hat{\beta}_0(\tau)\} \approx \frac{2}{(N_0+1)} \sigma_n^2 + \sigma_x^2 \cdot r_0(\tau) \quad (3.75)$$

This will give an estimate $\hat{\beta}_0$ with the mean value

$$E\{\hat{\beta}_0\} = \sigma_x^2 \left(1 + \frac{2}{(N_0+1)} \cdot \frac{\sigma_n^2}{\sigma_x^2}\right). \quad (3.76)$$

In situations where the signal-to-noise ratio is very low, i.e. $\sigma_n^2/\sigma_x^2 \gg 1$, one requires a very large value of N_0 . Since the inequality $T \gg T_D$ is also required this causes a very long estimation time $N_0 T$.

To cope with this one has to introduce pauses, $T_p > T_D$, and cancel the bias by using the estimating function

$$\hat{\beta}_{T_p}'(\tau) = \frac{N_0}{N_0-1} \cdot \left[\frac{2}{(N_0+1)(T-T_p)} \int_0^{N_0 T} y^2(t) q_{T_p}(t-\tau) d\tau - \frac{2}{N_0+1} \cdot \frac{1}{N_0 T} \int_0^{N_0 T} y^2(t) dt \right] \quad (3.77)$$

This gives

$$\max_{0 \leq \tau < N_0 T} E\{\hat{\beta}_{T_p}'(\tau)\} = \sigma_x^2 \quad (3.78)$$

and thereby an unbiased estimate. In order also to estimate σ_n^2 we use the results from section 3.3.1 above giving

$$\hat{\alpha}_{T_p}'(\tau) = \hat{\xi} - \frac{(N_0+1)}{2N_0} \hat{\beta}_{T_p}'(\tau) \quad (3.79)$$

where

$$\hat{\xi} = \frac{1}{N_0 T} \int_0^{N_0 T} y^2(t) dt. \quad (3.80)$$

The function $E\{\hat{\beta}_{T_p}'(\tau)\}$ is in point of principal plotted in Fig. 3.4.

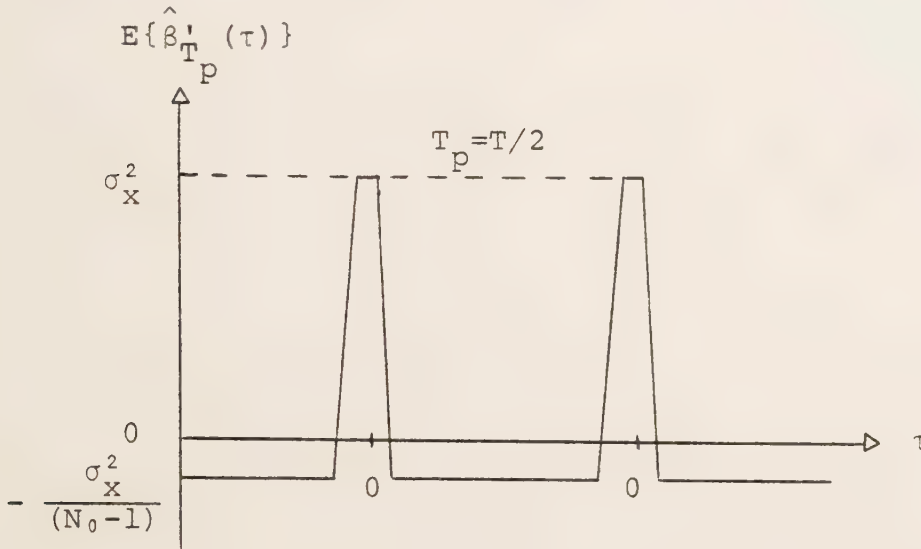


Figure 3.4. The function $E\{\hat{\beta}_{T_p}'(\tau)\}$ in point of principal.

The method, which uses a pseudorandom pattern signal, is well suited for outdoor measurements. This is due to the fact that well separated reflections, common outdoors, can be measured separately.

4. ON THE DESIGN OF A DIGITAL POWER CORRELATOR

A digital power correlator according to chapter 2 and 3 will be described. In this chapter the design is outlined and specifications given. The design was made in close cooperation with B. Bengtsson and in [9] this design is reported giving more details.

There are several reasons why to choose a discrete time and digital realization. One is the requirement of large dynamic range. The task is therefore to design the system shown in Fig. 4.1.

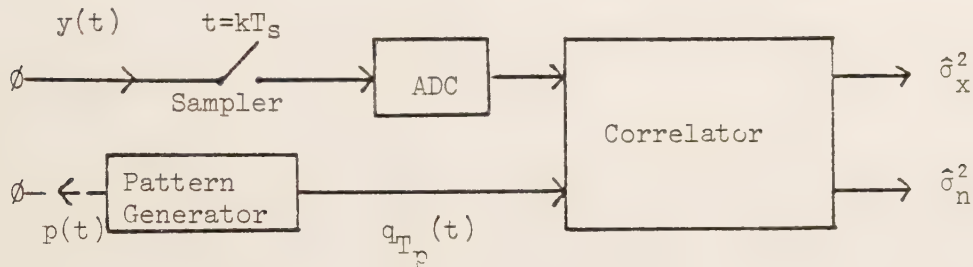


Figure 4.1. The digital estimation system.

The estimate $\hat{\sigma}_n$ is calculated using $\hat{\sigma}_x$ and $\hat{\xi}$ according to section 3.3.1 and 3.4.

The following parameters have to be considered:

- number of bits of the ADC
- sampling rate, f_s
- on-off time, T
- resolution, T_c
- number of points of the estimating function, K_{T_0}
- estimation time, $N_p T_0$.

Furthermore we also have to design a pseudo-noise generator.

The requirement of large dynamic range, 50 dB, is satisfied using an ADC with 12 bits.

The bandwidth of acoustic systems of interest is limited to 20 kHz. This means that a sampling rate of 100 kHz is sufficient. The lowest sampling rate, possible to reach, is 10 Hz (infra sonics).

It shall be possible to choose the on-off time between 0.1 and 100 s.

The estimating function will be presented graphically and studied visually. One hundred points will then be sufficient.

The maximum estimation time, $N_p T_0$, that is possible to reach, shall be 1000 s.

The correlator gives the estimation function in a number of points, i.e. $\{\hat{\beta}_{T_p}(kT_0)\}_{k=1}^{KT_0}$.

A resolution, T_c , less than 0.1 s is not of interest. The resolution also means that squared samples are accumulated into groups. These groups constitute the input signal to the correlator. The number of samples in one group is

$$L = f_s \cdot T_c \quad (4.1)$$

Before going further we present the blockdiagram of the digital power correlator in Fig. 4.2.

In Fig. 4.2 PER denotes on-off pattern according to chapter 2 and Skip K_s and Skip M_s are delays of $L \cdot K_s$ and $L \cdot K \cdot M_s$ number of samples respectively. The blocks shown in Fig. 4.2 are all realized in hardware. The correlator that correlates y_k and $q_{T_p}(t)$ is realized by means of an ABC 80 computer. The details and motives are presented in [9].

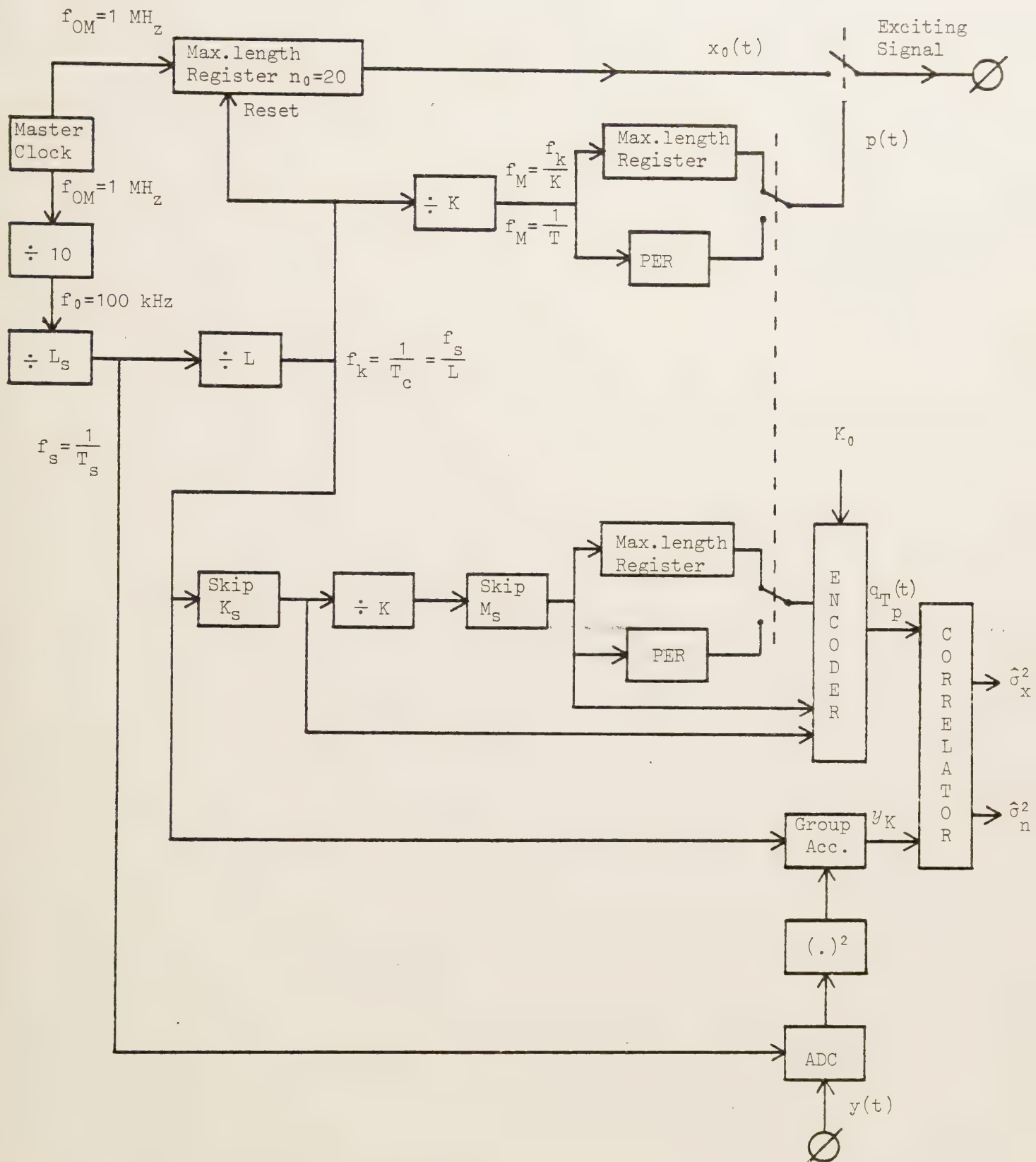


Figure 4.2. Blockdiagram of the digital estimation system.

The following relations are introduced in Fig. 4.2:

$$f_K = \frac{1}{T_C} = \frac{f_S}{L} = \frac{1}{LT_S} , \quad (4.2)$$

$$f_M = \frac{1}{T} = \frac{f_K}{K} \quad (4.3)$$

and

$$T_P = K_0 T_C \quad (4.4)$$

This means that

$$K > K_0 \quad (4.5)$$

We note that

$$T = KLT_S \quad (4.6)$$

requires that KL values of up to 10^7 are possible.

Since a K greater than 100 is meaningless, we propose that it shall be possible to choose any K and L such that

$$1 \leq K \leq 100 \quad (4.7)$$

and

$$1 \leq L \leq 10^5 .$$

The ADC comprises 12 bits and thereby the squared samples 22 bits. Since L may be 10^5 one would have 39 or 38 bits in the group accumulator. However it was very favourable to choose 35 bits, [9], and we decided to do so. This guarantees no overflow for $L < 8192$, in practice for $L < 10\,000$.

The maximum-length generator that gives $x_0(t)$ has a period length of $2^{20} - 1 = 1\,048\,575$ samples and in terms of time that is 1.048575 s. This will give a frequency spectrum

with a resolution of 1 Hz. In most applications this is sufficient. The register is reset with the frequency f_k in order to give an equal signal $x_0(t)$ for each time interval T_c .

5. PRACTICAL MEASUREMENTS

We will here present a few experiments made using the digital power correlator. The signal $x_0(t)$, pattern modulated, excites the room by means of a loudspeaker. There is also another loudspeaker exciting the room with background noise. The exciting signals were wideband signals. From a microphone in the room the signal $y(t)$ was obtained. In order to check the estimates the sound levels were measured without background and compared to the estimates.

In Fig 5.1 the estimating function $\hat{\beta}'_{T_p}$ is shown for an experiment made on the conditions:

Background noise level: Negligible
 $N_0 = 7$ $K = 12 \Rightarrow T = 14.4 \text{ s}$
 $L = 12\ 000$ $K_0 = 6 \Rightarrow T = 7.2 \text{ s}$
 $f_s = 10 \text{ kHz}$

If T_p is large enough we shall get a maximum interval. The resolution of $\hat{\beta}'_{T_p}(\tau)$ is $T_c = 1.2 \text{ s}$.

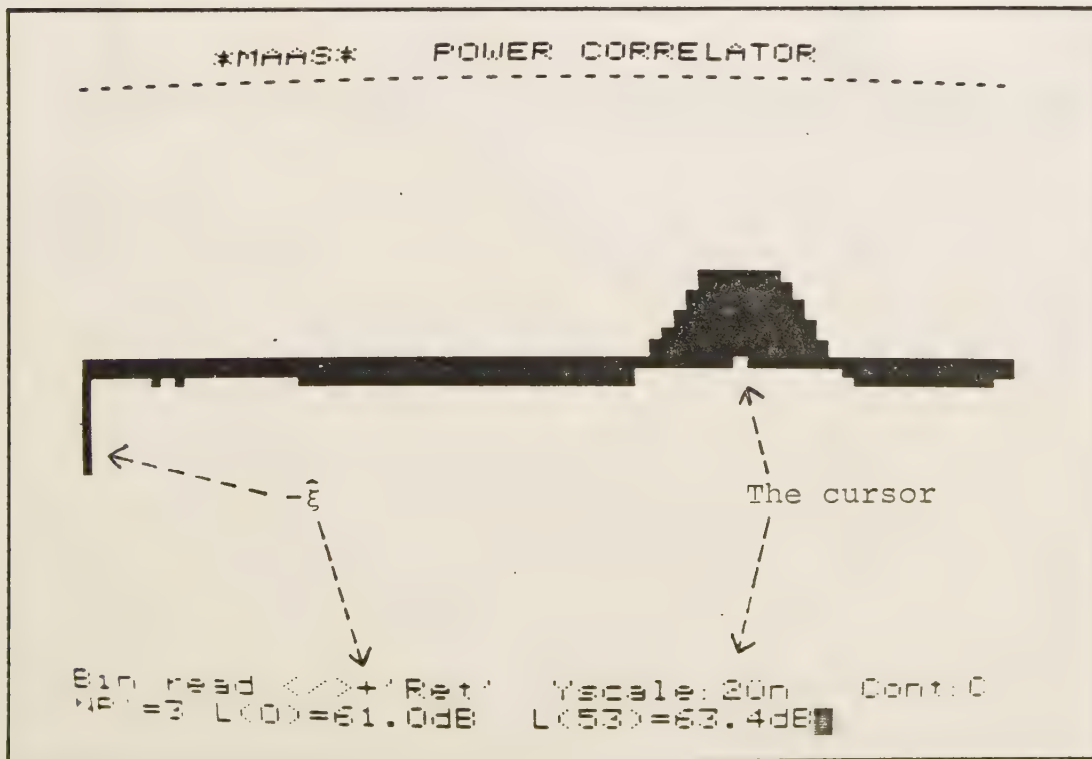


Figure 5.1. The function $\hat{\beta}'_{T_p}(\tau)$ without background noise and $T_p = 7.2 \text{ s}$, $T = 14.4 \text{ s}$.

The off-line measurement gave $\sigma_x^2 = 63.4$ dB and the digital power correlator gave $\sigma_x^2 = 63.4$ dB and as expected a maximum interval.

Using

$$\begin{aligned} N_0 &= 7 & K &= 4 \Rightarrow T = 7.52 \text{ s} \\ L &= 18\,000 & K_0 &= 2 \Rightarrow T_p = 3.76 \text{ s} \\ f_s &= 10 \text{ kHz} \end{aligned}$$

the estimating function given in Fig. 5.2 was found. The delay T_d is known to be within the interval $0 < T_d < 0.01$ s.

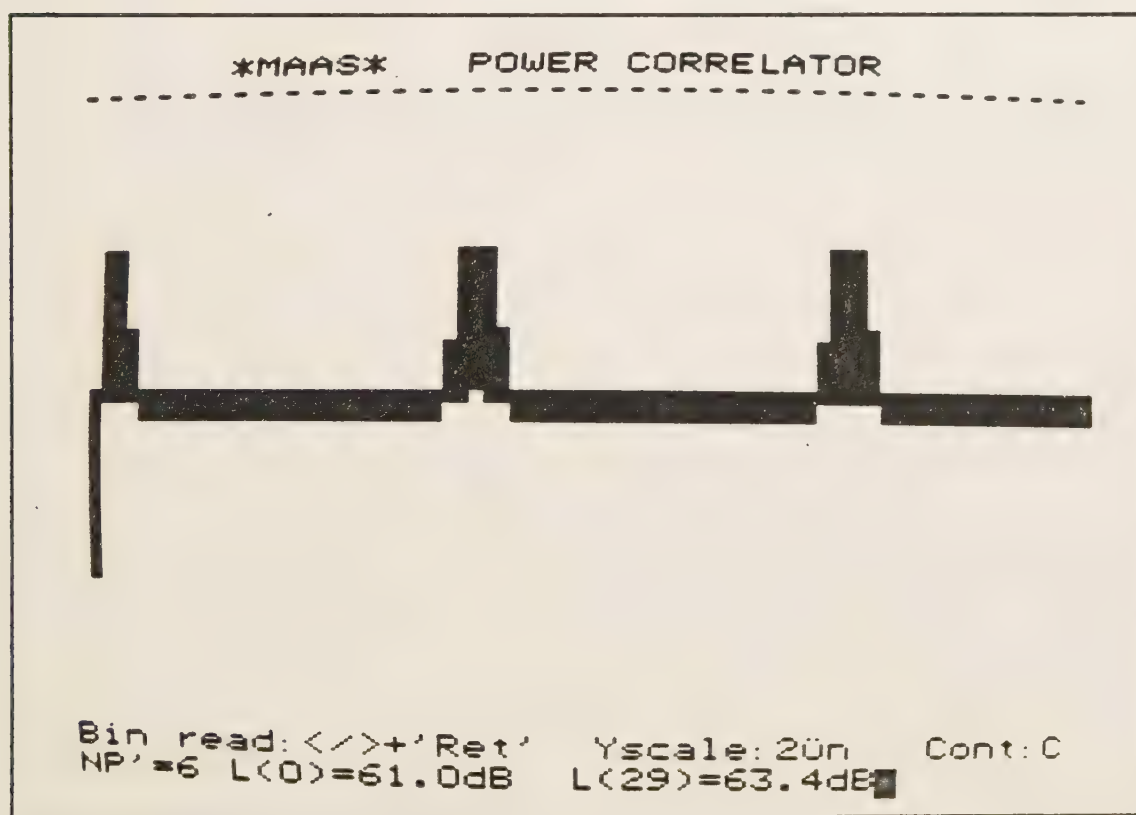


Figure 5.2. The function $\hat{\beta}'_{T_p}(\tau)$ without background noise, $T_p = 3.76$ s and $T = 7.52$ s.

The off-line measurements gave $\sigma_x^2 = 63.4$ dB and the digital correlator gave $\sigma_x^2 = 63.4$ dB and as expected a maximum interval.

In order to test if a pause time $T_p > 0$ is necessary we carried out the following experiment

$$\begin{aligned} N_0 &= 7 & K &= 1 \Rightarrow T = 1.88 \text{ s} \\ L &= 18\,000 & K_0 &= 0 \Rightarrow T_p = 0 \\ f_s &= 10 \text{ kHz} \end{aligned}$$

The estimating function $\hat{\beta}_0^1$ is given in Fig 5.3.

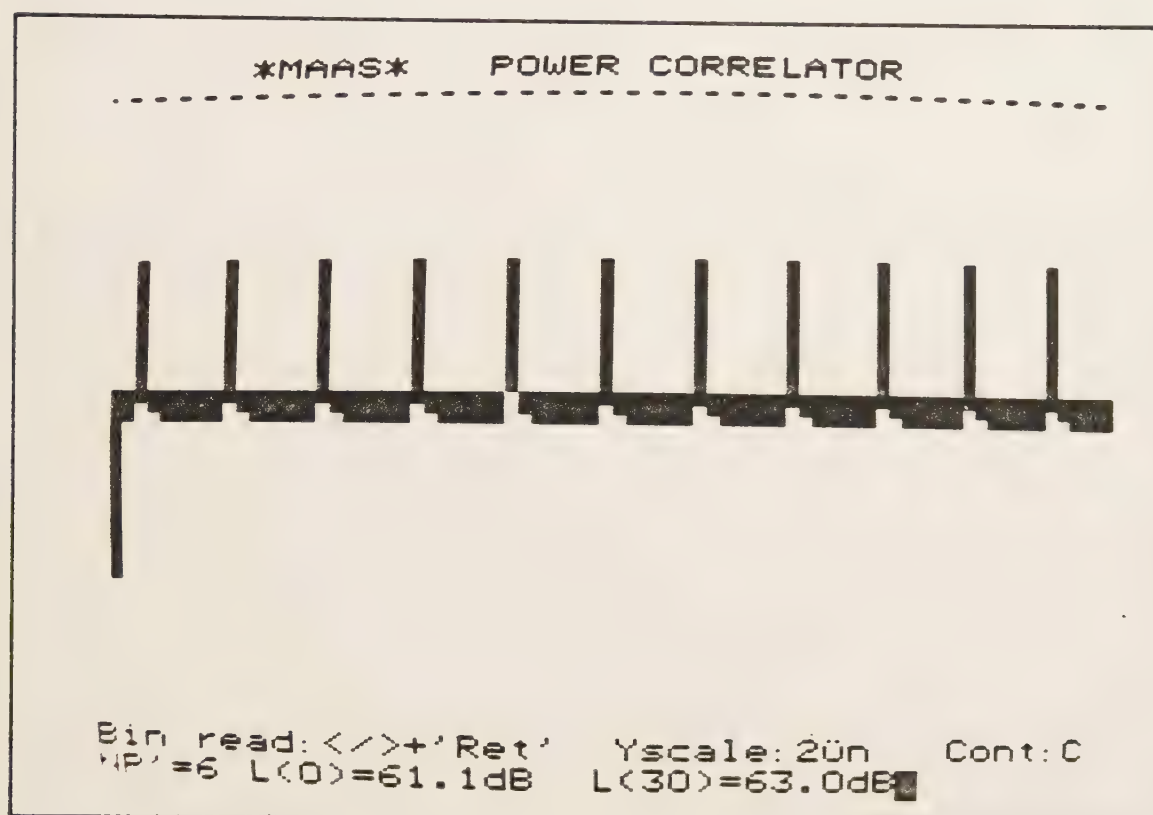


Figure 5.3. The function $\hat{\beta}_{T_p}^1(\tau)$ without background noise and $T_p = 0$, $T = 1.88$ s.

The off-line measurement gave $\sigma_x^2 = 63.4$ dB and the digital correlator gave $\sigma_x^2 = 63.0$ dB. This deviation is due to the fact that $T_p = 0$. (The delay is negligible, $T_d < 0.01$ s). The reverberation time of the experiment room was about 2 s. If we use (3.48) we find that an error of 0.7 dB is to be expected. The difference may be explained if the "effective" reverberation time is smaller than 2 s.

We have also carried out a few experiments with very high background noise. One is:

$$\begin{array}{ll} N_0 = 15 & K = 4 \Rightarrow T = 8 \text{ s} \\ L = 10\,000 & K_0 = 2 \Rightarrow T_P = 4 \text{ s} \\ f_s = 5 \text{ kHz} & N_P = 16 \end{array}$$

Where N_P denotes the number of periods $N_0 T$. This means that the total estimation time was $N_P \cdot N_0 T = 16 \cdot 15 \cdot 8 = 1920 \text{ s}$. The signal-to-noise ratio was

$$S_{NR} = -10 \text{ dB} \quad (5.1)$$

The off-line measurement gave $\sigma_x^2 = 70.2 \text{ dB}$ and the digital correlator gave $\hat{\sigma}_x^2 = 70.1 \text{ dB}$.

Experiments performed outdoors will be carried out at the end of the Summer 1981.

6. CONCLUSIONS

In this paper the task of measuring attenuation of a noisy acoustic channel has been studied. There are several acoustic applications where the background noise causes problems giving low signal-to-noise ratios. In order to cope with this, different signal processing techniques may be used. One is the coherence method, a well-known tool in acoustic applications. However this method can not in general be used in outdoor measurements. Since such measurements are here essential applications we have studied another signal processing method that has shown to be applicable even for outdoor acoustic channels.

This method uses an on-off modulation of the exciting signal such that the power of the exciting signal, measured in a point, can be estimated. The method is suggested in [3] and has in this paper been analysed and developed.

The power of the background noise is also often of interest. A discrete time method estimating signal and background noise power is deduced and found to be efficient on certain assumptions. A corresponding analog time method is also studied and found to be equivalent to the discrete time method for any reasonable measuring situation.

One assumption that is made for deducing the discrete time method is that independent on-off observations are received. This assumption can be met if pauses, T_p , are introduced to the signal processing. Let T_D be the duration of the impulse response of the system in view. Then it is found that, by calculating a cross-correlation function for $T_p > T_D$, an unbiased estimate of the signal power is given. The method gives unbiased estimates and the variances of these estimates are calculated.

The method suggested in [3] uses $T_p=0$. This means that each on and off time has to be long compared with T_D . If there is a delay of the system in view this delay can be estimated by using the cross-correlation function.

By using pauses, T_p , the duration of the impulse response is also estimated. This is due to the fact that for $T_p > T_D$ there exists a maximum interval of the cross-correlation of length $\Delta T = T_p - T_D$. The estimate of the signal power is well defined for any finite T_d and $T_D < T_p$.

An implementation of this estimation procedure is made giving estimates of signal power and background noise power. The design of this realization, a digital power correlator, is outlined. Introductory measurements using this power correlator is performed and presented.

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APPENDIX

We will here calculate the following two variances given in chapter 2:

$$V\{\hat{\beta}\} = \frac{2}{T^2} \int_0^{T_1} \int_0^{T_1} [r_x^2(t_1-t_2) + 2r_n(t_1-t_2)r_x(t_1-t_2) + r_n(t_1-t_2)] dt_1 dt_2 +$$

$$+ \frac{2}{T_2^2} \int_0^{T_2} \int_0^{T_2} r_n^2(t_1-t_2) dt_1 dt_2 \quad (A1)$$

and

$$V\{\hat{\alpha}\} = \frac{2}{T_2} \int_0^{T_2} \int_0^{T_2} r_n^2(t_1-t_2) dt_1 dt_2 \quad (A2)$$

where $r_n(\tau)$ and $r_x(\tau)$ are covariance functions of the noise and the signal component of the received signal $y(t)$ respectively. Since the spectral densities of these signals are assumed to be constant over the frequency interval $f_1 < |f| < f_1+B$, the variance of $\hat{\beta}$ becomes

$$V\{\hat{\beta}\} = \frac{\sigma_x^4 + \sigma_n^4 + 2\sigma_n^2\sigma_x^2}{T_1} \int_{-T_1}^{T_1} 2\cos^2(2\pi f_c \tau) \frac{\sin^2 \pi B \tau}{(\pi B \tau)^2} \left(1 - \frac{|\tau|}{T_1}\right) d\tau +$$

$$+ \frac{\sigma_n^4}{T_2} \int_{-T_2}^{T_2} 2\cos^2(2\pi f_c \tau) \frac{\sin^2 \pi B \tau}{(\pi B \tau)^2} \left(1 - \frac{|\tau|}{T_2}\right) d\tau. \quad (A3)$$

where f_c is the centre frequency given by

$$f_c = f_1 + B/2. \quad (A4)$$

We first note that for $f_c = B/2$, that is lowpass, we have

$$V\{\hat{\beta}\} = \frac{\sigma_x^4 + \sigma_n^4 + 2\sigma_x^2\sigma_n^2}{T_1} 2 \int_{-T_1}^{T_1} \frac{\sin^2(2\pi B \tau)}{(2\pi B \tau)^2} \left(1 - \frac{|\tau|}{T_1}\right) d\tau +$$

$$+ \frac{\sigma_n^4}{T_2} 2 \int_{-T_2}^{T_2} \frac{\sin^2 2\pi B \tau}{(2\pi B \tau)^2} \left(1 - \frac{|\tau|}{T_2}\right) d\tau < \frac{(\sigma_x + \sigma_n)^2}{BT_1} + \frac{\sigma_n^4}{BT_2} \quad (A5)$$

An upper bound of the variance for $f_c \geq B/2$ is given by

$$V\{\hat{\beta}\} < \frac{\sigma_x^4 + \sigma_n^4 + 2\sigma_x^2\sigma_n^2}{BT_1} + \frac{\sigma_n^4}{BT_2} \quad (A6)$$

since for $f_c > B/2$

$$\begin{aligned} & \frac{2}{T} \int_{-T}^T \cos^2(2\pi f_c \tau) \frac{\sin^2(\pi B \tau)}{(\pi B \tau)^2} \left(1 - \frac{|\tau|}{T}\right) d\tau < \\ & < \frac{2}{T} \int_{-\infty}^{\infty} (\cos(2\pi f_c \tau) \frac{\sin(\pi B \tau)}{(\pi B \tau)})^2 d\tau = \frac{1}{BT} \end{aligned} \quad (A7)$$

We also want to find the asymptotical behaviour of the variance for large BT_1 and BT_2 . Since

$$\begin{aligned} & \frac{2}{T} \int_{-T}^T \cos^2(2\pi f_c \tau) \frac{\sin^2(\pi B \tau)}{(\pi B \tau)^2} \left(1 - \frac{|\tau|}{T}\right) d\tau = \\ & = \frac{2}{BT} \int_{-BT}^{BT} \cos^2\left(2\pi \frac{f_c}{B} \tau\right) \frac{\sin^2(\pi \tau)}{(\pi \tau)^2} d\tau - \\ & - \frac{4}{(BT)^2} \int_0^{BT} \tau \cos^2\left(2\pi \frac{f_c}{B} \tau\right) \frac{\sin^2(\pi \tau)}{(\pi \tau)^2} d\tau = \frac{1}{BT} (1 - \varepsilon(BT)) \end{aligned} \quad (A8)$$

where

$$0 < \varepsilon(BT) < \frac{4}{(BT)} \left[\frac{1}{\pi^2} + \frac{1}{2} + \frac{\ln(BT)}{\pi^2} \right], \text{ for } BT \geq 1, \quad (A9)$$

we note that the variance is given by

$$V\{\hat{\beta}\} = \frac{\sigma_x^4 + \sigma_n^4 + 2\sigma_x^2\sigma_n^2}{BT_1} (1 - \varepsilon(BT_1)) + \frac{\sigma_n^4}{BT_2} (1 - \varepsilon(BT_2)). \quad (A10)$$

This means that for $BT_1 \gg 1$ and $BT_2 \gg 1$ we have

$$V\{\hat{\beta}\} \approx \frac{\sigma_x^4 + \sigma_n^4 + 2\sigma_n^2\sigma_x^2}{BT_1} + \frac{\sigma_n^4}{BT_2} \quad (A11)$$

and

$$V\{\hat{\alpha}\} \approx \frac{\sigma_n^4}{BT_2} . \quad (A12)$$

The latter formula is derived by making the corresponding calculations.

